

Quantum information processing via a lossy bus

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We describe a method to perform two qubit measurements and logic operations on pairs of qubits which each interact with a harmonic oscillator degree of freedom (the *bus*), but do not directly interact with one another. Our scheme uses only weak interactions between the qubit and the bus, homodyne measurements, and single qubit operations. In contrast to earlier schemes, the technique presented here is extremely robust to photon loss in the bus mode, and can function with high fidelity even when the rate of photon loss is comparable to the strength of the qubit-bus coupling.

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The practical implementation of quantum information processing (QIP) devices is one of the central aims of quantum information science. Large scale quantum computers can dramatically outperform classical computers for certain problems [1], while few qubit devices would allow other useful QIP applications, including quantum repeaters [2] and cryptography [3].

Of particular interest are *bus mediated* QIP architectures, in which coherent quantum operations on multiple qubits are implemented indirectly, via a continuous degree of freedom (the *bus*) such as a quantized electromagnetic field mode which interacts with each of the qubits. Such architectures have a number of potential advantages for practical implementations. Firstly, in some systems, engineering a coherent interaction between qubits and a bus mode can be easier than directly coupling pairs of qubits. Secondly, architectures involving a bus mode can be more readily scaled to large numbers of qubits. When a new qubit is added to the system, only its interaction with the bus mode needs to be calibrated and controlled. Bus mediated operations can then be implemented on *any* pair of qubits, rather than being restricted to (say) nearest neighbour qubits. Finally, in many cases the bus can be propagated long distances, which can be useful for distributed applications such as quantum repeaters.

Recently, additional interest in bus-mediated QIP has been generated by several experiments which have demonstrated the coherent coupling between qubits and continuous degrees of freedom. These experiments include the strong coupling of a superconducting charge qubits to a single electromagnetic field mode [4], the coupling of trapped ion qubits to a collective vibrational mode [5] and the observation of strong coupling in various quantum dot-cavity and atom-cavity systems [6, 7, 8, 9].

Previously, one of us introduced a scheme for implementing two-qubit operations via a bus mode [10]. This was described in terms of an all-optical implementation, using a weak cross-Kerr interaction between the photonic

qubits and a bus which is a single mode of the optical field, though the scheme could be modified to work for other systems with a suitable qubit-bus coupling Hamiltonian. In this scheme, the bus mode is initially prepared in a coherent state $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \alpha^n |n\rangle / \sqrt{n!}$, where $|n\rangle$ denotes the n 'th excited state of the oscillator. Subsequently the bus interacts with the qubit degrees of freedom via appropriately chosen cross-Kerr interactions. Finally, by performing a homodyne measurement on the bus degree of freedom, the bus is disentangled from the qubits, and the entanglement is transferred to the qubits. In the original proposal [10], this technique was used to implement two-qubit entangling measurements on the qubits, which are sufficient for universal quantum computation [11]. A modification of the scheme also allows the implementation of a CNOT gate [12]. Other schemes for implementing gates using a bus mode have also been proposed [13, 14].

However, these schemes have a substantial drawback with regard to practical implementations. At various stages in the protocol, the bus mode is in a superposition of coherent states, $|\alpha e^{i\theta_q}\rangle$, with distinct phases $\{\theta_q\}$ which correspond to different states of the qubits. Superpositions of this form are rather fragile in the presence of dissipation (e.g. photon loss) of the bus degree of freedom. Hence, losing even a small number of quanta from the bus can lead to complete decoherence of the combined qubit-bus state, and destroy any entanglement between the qubits. A careful analysis shows that the scheme introduced in [10], and those derived from it [12], can only work in the regime of extremely low dissipation defined by $\kappa/\chi \ll \chi\tau$, where κ is the loss rate for quanta in the bus mode, χ is the interaction strength between the qubits and the bus, and τ is the corresponding interaction time. Since, in most realistic implementations, the bus mode will be subject to dissipation, this requirement can be a significant restriction.

In this Letter, we describe a method for bus mediated quantum information processing which is *robust to dissipation* of the bus mode. The physical resources used for this scheme are similar to those used in the original scheme [10], namely preparation of a coherent state of

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the bus mode, conditional phase shifts of the bus and homodyne measurements. In addition, the new scheme also makes use of unconditional *displacements* of the bus mode; such displacements are straightforward to implement in most systems, e.g. by driving the bus mode with a classical field resonant with the oscillator frequency. In particular, we show how, with these resources, a near-deterministic, two qubit projective parity measurement (i.e. a measurement of the operator $Z_1 Z_2$) can be implemented. Such a measurement, when augmented with single qubit operations, is universal for quantum computation, either using the CNOT construction of Ref. [15] or by constructing cluster states [16, 17]. The interaction between the qubits and the bus can be controlled in such a way that the scheme is robust to losses in the bus mode even in the regime $\kappa > \chi$, where the strength of the qubit-bus interaction is weak compared the loss rate.

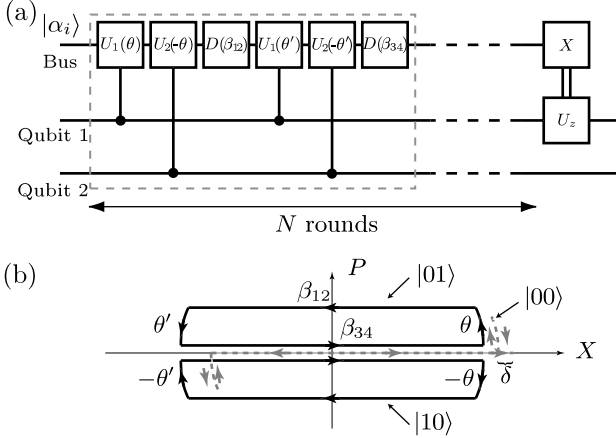


FIG. 1: The photon loss tolerant scheme. (a) The ‘circuit’ that implements the scheme. (b) A phase space illustration of a single round of the protocol. In the absence of dissipation, we would have $\theta' = -\theta$, $\beta_{12} = -2\alpha_i \cos \theta$ and $\beta_{34} = 2\alpha_i$. Dissipation leads to a slight deformation of the bus’ trajectory.

Our scheme for implementing parity measurements is illustrated in Fig. 1. The bus is prepared in the coherent state $|\alpha_i\rangle$, with α_i real. The bus then interacts with the qubits via repeated applications of the circuit shown in Fig. 1(a). The $U_j(\theta)$ operation is a conditional phase shift operation, which acts on qubit j and the bus mode as $|0\rangle|\alpha\rangle \rightarrow |0\rangle|\alpha e^{i\theta}\rangle$ and $|1\rangle|\alpha\rangle \rightarrow |1\rangle|\alpha e^{-i\theta}\rangle$ (throughout this Letter, the states are written with the ket for the bus mode at the right). $D(\beta) = e^{\beta a^\dagger - \beta^* a}$ is the displacement operator acting on the bus mode. The effect of a single round of the protocol can be understood by considering the phase space diagram in Fig. 1(b). For the 2-qubit input states $|01\rangle$ and $|10\rangle$, the bus mode follows the indicated loops in phase space. θ , θ' , β_{12} and β_{34} are chosen such that the bus returns its initial state, $|\alpha_i\rangle$, at the end of each round of the protocol. For the input states $|00\rangle$ and $|11\rangle$, the consecutive $U_j(\theta)$ operations cancel each other out, and the bus mode follows a

path along the x -axis. Since $\beta_{34} > \beta_{12}$, the final state of the bus, for the even parity states, is displaced by a small distance $\delta \approx \alpha_i \theta^2$. By repeating this procedure N times, the relative displacement of the bus mode for the odd and even parity qubit inputs can be made sufficiently large that the two cases can be distinguished by a homodyne measurement of the x -quadrature of the bus mode. Finally, if the measurement outcome indicates an odd parity state, a fixed rotation about the z -axis is applied to one of the qubits to correct for an unwanted phase accumulation during the protocol.

As we show below, by making θ sufficiently small, this procedure can be made extremely tolerant to photon loss. Heuristically, this works because, for a given input state parity, the bus mode is never in a superposition of coherent states with substantially different complex amplitudes. Thus, while a moderate amount of photon loss reduces the total amplitude of the bus state, it does not destroy the coherence of the superposed states within the relevant parity subspaces.

To analyze this scheme in more detail, we consider a pair of qubits coupled to a harmonic oscillator (the bus) via the interaction picture Hamiltonian ($\hbar = 1$)

$$H_I(t) = \frac{\chi_1(t)}{2} Z_1 a^\dagger a + \frac{\chi_2(t)}{2} Z_2 a^\dagger a + i\varepsilon(t)(a^\dagger - a). \quad (1)$$

Here, $\chi_i(t)$ denotes the strength of the coupling between qubit i and the bus mode, Z_i is the Pauli z -operator for the i th qubit, a is a lowering operator for the bus, and $\varepsilon(t)$ denotes the strength of the external driving field acting on the bus, which is resonant with the bare oscillator frequency [21]. We assume the phase and amplitude of this driving term can be accurately controlled. The first two terms in Eq. (1) generate the conditional phase shifts of the oscillator, while the third term generates the unconditional displacements of the oscillator. This Hamiltonian can be realized in a variety of settings, including the dispersive regime of cavity quantum electrodynamics (CQED) [22] (which can be realized in a variety of atomic and solid-state systems, and can also be simulated in an ion trap system [5]), and in an all-optical setting, via the cross-Kerr effect [10].

A single round of the scheme can be implemented by a piecewise constant time dependence for $\chi_i(t)$ and $\varepsilon(t)$. We take $\chi_1(t) = -\chi_2(t) = \{\chi_0, 0, -\chi_0, 0\}$, and $\varepsilon(t) = \{0, -\varepsilon_0, 0, \varepsilon_0\}$ for the intervals $\{0 < t < t_1, t_1 < t < t_2, t_2 < t < t_3, t_3 < t < t_4\}$. This has the effect of implementing the consecutive pairs of $U_j(\pm\theta)$ operations [shown in Fig. 1(a)] simultaneously, which is conceptually equivalent to performing them consecutively, since the Hamiltonian terms that generate these operations commute with one another. We also define the time intervals $\tau_i = t_i - t_{i-1}$ corresponding to the length of each step of the evolution. The results obtained using this choice for $\chi_i(t)$ and $\varepsilon(t)$ will be qualitatively valid for other choices, too. For example, in a particular implementation, it may be easier to have a fixed, constant qubit-bus coupling, and to periodically apply very strong,

short driving pulses (with $|\varepsilon| \gg |\chi_i|$) to the bus mode.

We describe dissipation using a standard approach for the CQED setting: the damping from the oscillator may be modeled as leakage from the corresponding cavity mirror at a rate 2κ . Within the Born, Markov, and rotating wave approximations, such losses can be described by the non-unitary term in a Lindblad master equation [18]:

$$\dot{\rho}(t) = -i[H_I(t), \rho(t)] + 2\kappa\mathcal{D}[a]\rho(t), \quad (2)$$

where $\rho(t)$ is the density matrix of the combined system of both qubits and the oscillator mode, and $\mathcal{D}[A]\rho = A\rho A^\dagger - (\rho A^\dagger A + A^\dagger A\rho)/2$ [23]. Below, we solve Eq. (2) for a single round of the protocol, and then use the results to determine both the total number of rounds required to distinguish even parity states from odd parity states, and the total dephasing error at the end of these N rounds.

The solution of Eq. (2) can be simplified by noting that, since the aim of the scheme is to perform a parity measurement, only dephasing errors *within* the corresponding parity subspaces are of concern. Thus we are only interested in solutions of Eq. (2) for initial states of the form $(x|00\rangle + y|11\rangle)|\alpha_i\rangle$ and $(x|10\rangle + y|01\rangle)|\alpha_i\rangle$, where x and y are arbitrary amplitudes. We take the worst case, $x = y = 1/\sqrt{2}$, and so the resulting errors should be viewed as an upper bound. The solution can be further simplified by projecting $\rho(t)$ onto the computational basis of the qubits. This results in a set of uncoupled differential equations for the operators $\rho_{i|j}(t) = \langle i|\rho(t)|j\rangle$, where $i, j \in \{00, 01, 10, 11\}$, which can be solved independently. These equations are analogous to those describing the single-qubit, driven Jaynes-Cummings model in the dispersive limit, and thus can be solved exactly using techniques similar to those used in Refs. [19, 20]. We omit the details of this calculation, but give the full analytic results here.

For an input state $|\Psi_{e+}\rangle|\alpha_i\rangle$, where $|\Psi_{e\pm}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, the state after a single round of the protocol (i.e. at time t_4), is given by $|\Psi_{e+}\rangle|\alpha_e\rangle$, with

$$\alpha_e = \beta_{34} + e^{-\kappa(\tau_3+\tau_4)}(\beta_{12} + e^{-\kappa(\tau_1+\tau_2)}\alpha_i),$$

where $\beta_{12} = -(\epsilon_0/\kappa)[1 - e^{-\kappa\tau_2}]$, $\beta_{34} = (\epsilon_0/\kappa)[1 - e^{-\kappa\tau_4}]$, and the time intervals $\tau_i = t_i - t_{i-1}$ correspond to the length of each step of the evolution.

For an input state $|\Psi_{o+}\rangle|\alpha_i\rangle$, where $|\Psi_{o\pm}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$, the state after a single round is

$$\frac{1}{2} [(|01\rangle\langle 01| + |10\rangle\langle 10|) \otimes |\alpha_o\rangle\langle \alpha_o| + \dots \\ (e^{-\Gamma_{01}-\Gamma_{12}-\Phi_{12}-\Gamma_{23}}|01\rangle\langle 10| \otimes |\alpha_o\rangle\langle \alpha_o^*| + \text{h.c.})], \quad (3)$$

where

$$\alpha_o = \beta_{34} + e^{-i\chi\tau_3}e^{-\kappa(\tau_3+\tau_4)}(\beta_{12} + e^{-\kappa(\tau_1+\tau_2)}e^{i\chi\tau_1}\alpha_i),$$

and $\Gamma_{01} = \alpha_i^2[(1 - e^{-2\kappa\tau_1}) + \frac{\kappa}{\kappa-i\chi}(e^{-2(\kappa-i\chi)\tau_1} - 1)]$, $\Gamma_{12} = \alpha_i^2e^{-2\kappa\tau_1}(1 - e^{-2\kappa\tau_2})(1 - e^{2i\chi\tau_1})$, $\Phi_{12} = 2i\text{Im}(\alpha_i\beta_{12}e^{-\kappa(\tau_2+\tau_1)}e^{i\chi\tau_1})$, and $\Gamma_{23} = |\alpha_2|^2(1 -$

$e^{-2\kappa\tau_3}) + \alpha_2^2\frac{\kappa}{\kappa-i\chi}(e^{-2(\kappa-i\chi)\tau_1} - 1)$, with $\alpha_2 = \beta_{12} + e^{-\kappa(\tau_1+\tau_2)}e^{i\chi\tau_1}\alpha_i$. The real part of the exponent $-\Gamma_{01} - \Gamma_{12} - \Phi_{12} - \Gamma_{23}$ corresponds to a dephasing error (see below), while the imaginary part leads to a deterministic phase rotation which is ultimately corrected by applying the single qubit unitary U_z [see Fig. 1 (a)], after the homodyne measurement stage of the protocol.

The timescales τ_2 , τ_3 and τ_4 are chosen as follows. τ_2 and τ_3 can be fixed requiring that, for symmetry, at time t_3 , the bus mode associated with the input state $|\Psi_{o\pm}\rangle$ is in the state $\alpha_3 = -\alpha_i$. This leads to the approximate values $\tau_2 \approx (2 - \kappa^2\tau_1^2 - \chi^2\tau_1^2)\alpha_i/\epsilon_0$ and $\tau_3 \approx \tau_1(1 - 2\kappa\tau_1)$. τ_4 is chosen such that $\alpha_o = \alpha_i$, i.e. that the bus mode associated with the input state $|\Psi_{o\pm}\rangle$ is returned to its initial state at the end of each round of the protocol.

Using these values of τ_i , it is possible to obtain approximate expressions for the total dephasing error at the very end of the protocol. After N rounds, the state corresponding to an initial state $|\Psi_{o+}\rangle$ is $\rho_o^{(N)} = [(1-p)|\tilde{\Psi}_{o+}\rangle\langle\tilde{\Psi}_{o+}| + p|\tilde{\Psi}_{o-}\rangle\langle\tilde{\Psi}_{o-}|] \otimes |\alpha_i\rangle\langle\alpha_i|$, where $p = (1 - e^{-N\text{Re}(\Gamma_{01}+\Gamma_{12}+\Gamma_{23})})/2$ is the probability of a dephasing error, and $|\tilde{\Psi}_{o\pm}\rangle$ corresponds to $|\Psi_{o\pm}\rangle$, up to the single qubit correction U_z discussed above. The final state corresponding to an initial state $|\Psi_{e+}\rangle$ is $\rho_e^{(N)} = |\Psi_{e+}\rangle\langle\Psi_{e+}| \otimes |\alpha_e^{(N)}\rangle\langle\alpha_e^{(N)}|$, where $\alpha_e^{(N)} \approx \alpha_i + N\alpha_i\chi^2\tau_1^2$.

Writing the expression for the dephasing error in terms of τ_1 gives the approximate expression $p \approx 4N\alpha_i^2\kappa\chi^2\tau_1^3/3 + 4N\alpha_i^3\kappa\chi^2\tau_1^2/\epsilon_0$. The first term in this expression originates from decoherence on the ‘rotation’ parts of the evolution (i.e. $t_0 \rightarrow t_1$ and $t_2 \rightarrow t_3$), whereas the second term originates from the first ‘displacement’ part ($t_1 \rightarrow t_2$). Note that no decoherence results from the second displacement stage ($t_3 \rightarrow t_4$), as the bus mode is not in a superposition state during these parts of the evolution.

The total number of required rounds of the protocol, N , is fixed by the requirement that $|\alpha_e^{(N)}\rangle$ and $|\alpha_i\rangle$ are distinguishable by a X -homodyne measurement at the end of the protocol. For coherent states, this implies that $\alpha_e^{(N)} - \alpha_i = A$, where $A \gtrsim 1$ is a constant that depends on the required error in distinguishing the states $|\Psi_{o\pm}\rangle$ from the states $|\Psi_{e\pm}\rangle$. This requirement gives $N = A/\alpha_0\chi^2\tau_1^2$. Thus the total error is well approximated by

$$p \approx \frac{4A\alpha_i\kappa\tau_1}{3} + \frac{4A\alpha_i^2\kappa}{\epsilon_0}. \quad (4)$$

The first term in this expression can be made arbitrarily small by decreasing the value of $\kappa\tau_1$. The second term in this expression is proportional the ratio κ/ϵ_0 . Thus errors can be minimized provided the strength of the classical driving field can be made large compared to the photon loss rate. An important point to note is that both terms are independent of the ratio κ/χ , and therefore the scheme is robust to losses in the bus mode even in the regime $\kappa > \chi$, where the strength of the qubit-bus interaction is weak compared to the loss rate. The ap-

proximate error, together with the exact values for the same values of the parameters τ_i , are plotted in Fig. 2.

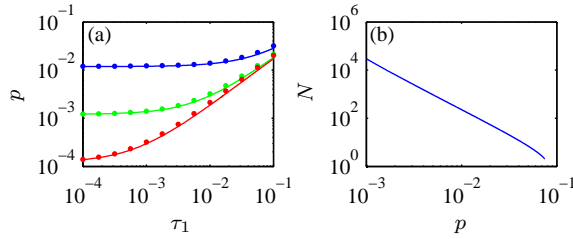


FIG. 2: (a) Exact (curves) and approximate (points) values of the final dephasing error against τ_1 . Other parameters: $A = 1.5$, $\alpha_i = 2$, $\kappa = 0.05$, $\chi = 1$, $\epsilon_0 = 10^2, 10^3, 10^4$, (top to bottom). (b) Overhead, N , required to reach a given error rate, p , for $A = 1.5$, $\alpha_i = 2$, $\kappa = 0.05$, $\chi = 1$, $\epsilon_0 = 10^5$.

The price to be paid for this robustness is an increase in the overhead cost of the scheme: the first term in Eq. (4) can be reduced by reducing τ_1 , but as τ_1 is reduced, the required number of rounds of the protocol required increases. If p is dominated by the first term, then the required number of rounds is $N \approx (16A^3\alpha_i/9)(\kappa/\chi)^2/p^2$. Thus the overhead cost grows quadratically with the required reduction in the errors (see Fig. (2)).

In view of this overhead, it is worth briefly considering the effect of errors in the scheme. Stochastic noise in the control pulses used to implement the protocol tend to broaden final state of the bus mode, making the odd and even parity outcomes harder to distinguish by the X -homodyne measurement. Treating the noise in each pulse as independent Gaussian processes, the final uncertainty in the real part of $\alpha_0^{(N)}$ is found to be $\sigma^2(\alpha_0^{(N)}) =$

$N\sigma^2(\alpha_0) \approx N[\sigma^2(\beta_{12}) + \sigma^2(\beta_{34}) + \beta_{12}^2 \sin^2 \theta \sigma^2(\theta)] \approx 2N\sigma^2(\beta_{12}) + 4\alpha_0 A \sigma^2(\theta)$, where $\sigma^2(\dots)$ denotes the variance in each parameter. Thus the noise will not significantly affect the outcome of the X -homodyne measurement provided $\sigma(\beta_{12}) \lesssim N^{-\frac{1}{2}}$ and $\sigma(\theta) \lesssim (4\alpha_0 A)^{-\frac{1}{2}}$. If these conditions cannot be met then the scheme can still be made to work by increasing the number of rounds, since the peak separation grows linearly in N , while the peak width grows only as $N^{\frac{1}{2}}$. Another potentially important source of noise is single qubit errors. These can potentially build up during the implementation of the two-qubit operation, leading to an effective error rate that scales with N . Thus our scheme is particularly suited to systems in which the single qubit decoherence rate is small compared to the bus damping rate. Fortunately, there are many systems (such as trapped ions [5]) where the single qubit error rate is much longer than all other timescales.

In conclusion, we have proposed a method for implementing two-qubit operations via a quantum bus, which is extremely robust to dissipation of the bus degree of freedom. Our method implements a parity measurement of the two-qubit system, which, when augmented with single qubit operations, is sufficient for universal computation. The residual errors in our scheme can be made negligible even when the strength of the qubit-bus interaction is weak compared to the loss rate. We anticipate that this robustness will permit bus-mediated QIP in a variety of physical setups, including superconducting systems, trapped ions or atoms, and all-optical systems.

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- [21] Note Eq. (1) is defined in an interaction picture relative to the bare Hamiltonian of the uncoupled qubits and bus, $H_0 = H_{q1} + H_{q2} + H_{\text{bus}}$, where $H_{qi} = -\frac{E_i}{2}Z_i$ and $H_{\text{bus}} = \omega_0 a^\dagger a$, with E_i the qubit energy splittings, and ω_0 the bare oscillator frequency. In arriving at Eq. (1), we have made a rotating wave approximation for the driving term, which is valid provided $\omega_0 \gg \varepsilon(t)$, and provided $\varepsilon(t)$ changes slowly on timescales of order ω_0^{-1} .
- [22] In a CQED implementation, additional multi-qubit terms can appear in the Hamiltonian due to higher order interactions via the bus. These can be avoided by engineering the system parameters such that $\text{sgn}(\Delta_1) = -\text{sgn}(\Delta_2)$, $g_1 \neq g_2$, and $g_{1,2} \lesssim |\Delta_1 + \Delta_2|$ where g_i are the qubit-cavity coupling strengths, and Δ_i are the qubit-cavity detunings.
- [23] Equation (2) describes errors only due to damping of the oscillator mode; in general, there will also be decoherence processes acting directly on the qubit degrees of freedom. However, such processes are typically independent of those due to damping and can be dealt with using standard fault tolerance techniques, and so we omit them from this analysis.